**Problem-Solving Strategies and Mathematical Reasoning**

**1.1 Problem Solving**

People often say, “I have a problem,” when they are faced with an unfamiliar or difficult situation and there is no obvious way to find an answer. Problem solvingis the process by which this new situation is analyzed and resolved. It begins with an understanding of all the aspects of the problem and ends when a satisfactory answer has been found.

This handout will help improve your skill at solving problems. Often, there is more than one way to approach a problem, and not every problem will be solved on the first try. The four basic steps that follow can help you to examine a problem situation.

**Step 1 Understand the problem.**

Read the problem carefully. Be sure you understand the situation and the question being asked. What information are you given? Are any facts missing or extra? Is there any special vocabulary? Look for important clues that may lead to a solution.

**Step 2 Formulate a plan for a solution.**

Organize the given data by using notes, lists, tables, and diagrams. Identify the mathematical operations that you will need to perform. If the plan involves many steps, list them in order. Estimate an answer if possible.

**Step 3 Solve the problem.**

Carry out the steps in your plan. Record the work for each step because it may be necessary to review and revise these steps at a later time.

**Step 4 Check the solution.**

Check your calculations. Did you make any errors? Does your answer make sense? Has the question been answered properly? If not, ask yourself what is wrong and how it can be fixed. If your check shows that you have successfully solved the problem, clearly identify the answer.

**1.2 Identifying Information**

To solve some problems, you must select the information that is needed from a source that may also contain extra information. Being able to recognize the unnecessary information will make you a more efficient problem solver. On the other hand, if a problem does not provide sufficient information, you will not be able to solve it unless you can find the missing facts. Sometimes, information may not be stated outright, but you are expected to know it. For example, you might be expected to know that there are 12 items in a dozen, 4 aces in a standard deck of cards, 365 days in 1 year, or 24 hours in a day.

**1.3 Guessing and Checking**

One way to begin working on a problem is to make a reasonable guess at the answer. You must then test the guess to see if it satisfies the conditions of the problem. If it does not, observing whether the guess is too high or too low helps you to make a better second guess. Continue guessing and checking until you find the correct answer.

**1.4 Working Backward**

Some problems tell you a final result and the steps it took to get there. These problems ask you to figure out what the starting point must have been. Working backward is a strategy that works well for this type of problem. For example, a problem might tell you that after going to the store and buying milk for $2, Tony had $3 in his wallet. The problem asks you how much money Tony must have had *before* he bought the milk. Working backward, you use the opposite or inverse of the operation that was performed when Tony bought the milk. Since spending $2 is subtraction, you do the inverse operationand add $2 back to the $3. You have figured out that Tony must have started with $5.

**1.5 Using Pictures and Diagrams**

When problems involve physical situations, the strategy of drawing a picture or diagramcan help you visualize the relationships in the problem. Once you have a better understanding of the problem, you can begin to plan a solution.

**1.6 Using Tables and Lists**

Tables and lists are useful for organizing information that relates to a problem. The entries displayed on a table can be examined and compared with the conditions of a problem. The solution to a problem may be one, some, or all of the entries in the table. Making a table or list may be combined with another strategy, such as guessing and checking or drawing a diagram.

**1.7 Recognizing Patterns**

When you are trying to identify a pattern, the way to begin is by examining the first few numbers, figures, or other parts of the problem. Once you recognize the pattern, you can predict other outcomes.

Some patterns are very simple, such as a series of numbers or figures that repeats. For example, the pattern below is triangle, square, circle, then repeat. So the next figure would be a square.

∆□○∆□○∆?

Other patterns are more complicated. A good way to begin is by figuring out what has changed from one term to the next. Ask yourself if the pattern is increasing or decreasing and by how much.

**1.8 Solving a Simpler Related Problem**

Sometimes you can find the answer to a problem by solving another problem that has simpler numbers or fewer cases. Using smaller or easier numbers, like whole numbers instead of decimals or fractions, can help you focus on the operations that are needed to plan a solution. Or, by starting with fewer cases and working up, you may be able to identify a pattern that will lead to a solution.

**1.9 Using Logical Reasoning**

**Logic** is the study of reasoning. All problem solving, whether in mathematics or daily life, involves reasoning, but there are some problems for which special approaches are very effective. These problems generally present several facts that can be used together to reach a conclusion. Combining reasoning skills with other strategies, such as drawing a diagram or making a table, will help you find the desired solution.

**1.10 Using Estimation**

An estimatetells you about how much. In later sections, you will learn and apply several estimation strategies. The idea behind all estimation strategies is to use numbers close to the numbers in the problem so that you can work with them mentally. Knowing when to estimate is an important skill in itself.

You can estimate when

* an exact answer is not needed.
* the problem can be solved just by knowing “about how much.”
* paper and pencil or a calculator is not available.
* you want to check if an answer is reasonable. A reasonable answer
* is one that makes sense for the given situation.

Once you decide to estimate, you may also need to think about whether an overestimate or underestimate would be more useful.

An overestimate is an estimate you know is greater than the exact answer. For example, to be sure that you take enough money, you may wish to overestimate the total cost of groceries.

An underestimate is an estimate you know is less than the exact answer. For example, you may want to underestimate the total amount of paint remaining in some started cans to be sure there is enough to finish a new job.

In some situations it does not matter if the estimate is high or low, such as when you are estimating the number of oranges on a tree.